Volatility Managed Portfolios

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What do we do?

1. Volatility managed portfolios: scale aggregate priced factor by $1/\sigma_t^2$

2. Motivation: risky asset demand

$$w_t = \frac{1}{\gamma}\frac{\mu_t}{\sigma_t^2}$$

3. Volatility doesn't forecast returns $\Rightarrow$ volatility timing beneficial

What do we find?

Volatility managed portfolios

1. increase Sharpe ratios, generate large alpha on original factors
2. take *less* risk in recessions when σ high
3. *sells* after market crashes (1929, 1987, 2008)

Outline

1. Vol managed portfolios empirically

2. Implications

Data

- Factors: Market, SMB, HML, Momentum, Profitability, ROE, Investment, Carry (FX)
- Daily and monthly data for each factor
- Sample: 1926-2015 (Mkt, SMB, HML, Momentum), Post 1960 for the rest
- All numbers annualized

Managed volatility factors

1. Let f_{t+1} be an excess return, construct

$$f^{\sigma}_{t+1} = \frac{c}{\sigma^2_t(f)} \times f_{t+1}$$

   - $\sigma_t(f)$ previous month realized volatility (daily data)
   - choose c so f^{σ} has same unconditional volatility as f

2. Regression:

$$f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$$

   We show:

$$\alpha = -cov\left(\frac{\mu_t}{\sigma^2_t}, \sigma^2_t\right) \frac{c}{E[\sigma^2_t]}$$

Moreira and Muir (2015)

Volatility managed factors: alphas

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}	(7) MVE^{σ}	(8) FX^{σ}	(9) ROE^{σ}	(10) IA^{σ}
MktRF	0.61									
	(0.05)									
SMB		0.62								
		(0.08)								
HML			0.57							
			(0.07)							
Mom				0.47						
				(0.07)						
RMW					0.62					
					(0.08)					
CMA						0.68				
						(0.05)				
MVE							0.58			
							(0.03)			
Carry								0.71		
								(0.08)		
ROE									0.63	
									(0.07)	
IA										0.68
										(0.05)
α	4.86	-0.58	1.97	12.51	2.44	0.38	4.12	2.78	5.48	1.55
	(1.56)	(0.91)	(1.02)	(1.71)	(0.83)	(0.67)	(0.77)	(1.49)	(0.97)	(0.67)
N	1,065	1,065	1,065	1,060	621	621	1,060	360	575	575
R2	0.37	0.38	0.32	0.22	0.38	0.46	0.33	0.51	0.40	0.47
rmse	51.39	30.44	34.92	50.37	20.16	17.55	25.34	21.78	23.69	16.58

Moreira and Muir (2015)

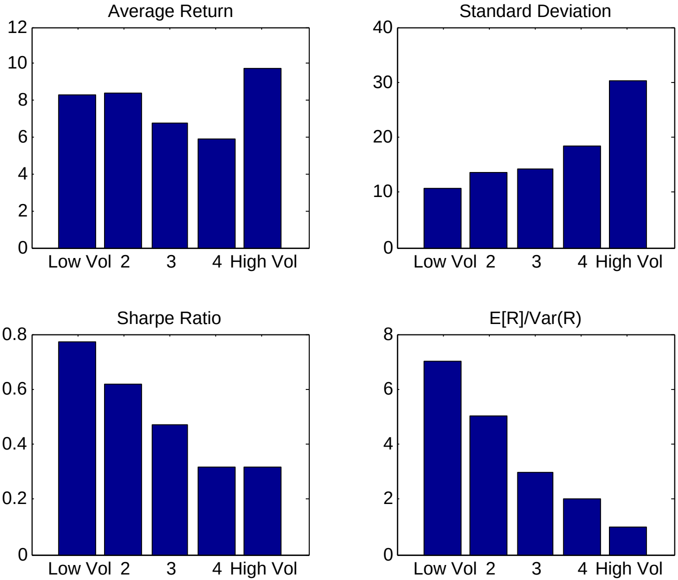
Volatility managed factors

How much do we increase Sharpe ratio / expand MVE frontier?

$$\frac{\alpha}{\sigma_\epsilon}$$

- MKT (0.33), HML (0.20), MOM (0.88), Profitability (0.41), Carry (0.44), ROE (0.80), Investment (0.32)

Vol timing works due to weak risk-return trade-off (market)



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Multiple factors

1. Some investors invest in multiple factors beyond the market
2. Extend our approach to the (static) MVE portfolio
   - For given set of factors construct in sample MVE: $f^*_{t+1} = b^* F_{t+1}$
   - Volatility time the MVE portfolio: $f^{\sigma,*}_{t+1} = \frac{c}{\sigma^2_t(f^*)} f^*_{t+1}$
   - Do this for alternative opportunity sets

MVE portfolios

	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom
Alpha (α)	4.86	4.99	4.04	1.34	2.01	2.32	2.51
	(1.56)	(1.00)	(0.57)	(0.32)	(0.39)	(0.38)	(0.44)
Observations	1,065	1,065	1,060	621	621	575	575
R-squared	0.37	0.22	0.25	0.42	0.40	0.46	0.43
rmse	51.39	34.50	20.27	8.28	9.11	8.80	9.55
Original Sharpe	0.42	0.52	0.98	1.19	1.34	1.57	1.57
Vol Managed Sharpe	0.51	0.69	1.09	1.20	1.42	1.69	1.73
Appraisal Ratio	0.33	0.50	0.69	0.56	0.77	0.91	0.91

1. MVE portfolios contain pricing information for a large cross-sectional of assets.
2. Appraisal/Sharpe $\approx$ 75%

Robustness / additional empirical results

1. Take less risk in recessions
2. Survive transactions costs
3. Leverage constraints
4. Works at longer horizons
5. Subsample results (weaker for 1956-1985)
6. Stronger results with expected vol
7. Look at 20 OECD equity indices
8. Multi-factor regressions: include BAB, risk-parity factors.
9. Well behaved higher moments
   - Generally, 10th and 1st percentiles of vol managed returns are above those for unconditional returns, look at skewness kurtosis also
10. Less exposed to variance risk and downside risk than original factors

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1. Vol managed portfolios take less risk in recessions

	(1) Mkt^{σ}	(2) HML^{σ}	(3) Mom^{σ}	(4) RMW^{σ}	(5) CMA^{σ}	(6) FX^{σ}	(7) ROE^{σ}	(8) IA^{σ}
MktRF	0.83							
	(0.08)							
MktRF $\times 1_{rec}$	-0.51							
	(0.10)							
HML		0.73						
		(0.06)						
HML $\times 1_{rec}$		-0.43						
		(0.11)						
Mom			0.74					
			(0.06)					
Mom $\times 1_{rec}$			-0.53					
			(0.08)					
RMW				0.63				
				(0.10)				
RMW $\times 1_{rec}$				-0.08				
				(0.13)				
CMA					0.77			
					(0.06)			
CMA $\times 1_{rec}$					-0.41			
					(0.07)			
Carry						0.73		
						(0.09)		
Carry $\times 1_{rec}$						-0.26		
						(0.15)		
ROE							0.74	
							(0.08)	
ROE $\times 1_{rec}$							-0.42	
							(0.11)	
IA								0.77
								(0.07)
IA $\times 1_{rec}$								-0.39
								(0.08)
Observations	1,065	1,065	1,060	621	621	362	575	575
R-squared	0.43	0.37	0.29	0.38	0.49	0.51	0.43	0.49

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2. Transaction costs

1. Results for the market portfolio
2. Transaction cost from Frazzini, Israel, and Moskowitz (2015)

					α After Trading Costs			
w	Description	$\mid \Delta w \mid$	$E[R]$	α	1bps	10bps	14bps	Break Even
$\frac{1}{RV_t^2}$	Realized Variance	0.73	9.47%	4.86%	4.77%	3.98%	3.63%	56bps
$\frac{1}{RV_t}$	Realized Vol	0.38	9.84%	3.85%	3.80%	3.39%	3.21%	84bps
$\frac{1}{E_t[RV_{t+1}^2]}$	Expected Variance	0.37	9.47%	3.30%	3.26%	2.86%	2.68%	74bps
$min\left(\frac{c}{RV_t^2}, 1\right)$	No Leverage	0.16	5.61%	2.12%	2.10%	1.93%	1.85%	110bps
$min\left(\frac{c}{RV_t^2}, 1.5\right)$	50% Leverage	0.16	7.18%	3.10%	3.08%	2.91%	2.83%	161bps

3. Leverage constraints

Volatility Timing and Leverage

Panel A: Weights and Performance for Alternative Volatility Managed Portfolios

w_t	Description	α	Sharpe	Appraisal	Distribution of Weights w: P50	P75	P90	P99
$\frac{1}{RV_t^2}$	Realized Variance	4.86 (1.56)	0.52	0.34	0.93	1.59	2.64	6.39
$\frac{1}{RV_t}$	Realized Volatility	3.30 (1.02)	0.53	0.33	1.23	1.61	2.08	3.36
$\frac{1}{E_t[RV_{t+1}^2]}$	Expected Variance	3.85 (1.36)	0.51	0.30	1.11	1.71	2.38	4.58
$min\left(\frac{c}{RV_t^2}, 1\right)$	No Leverage	2.12 (0.71)	0.52	0.30	0.93	1	1	1
$min\left(\frac{c}{RV_t^2}, 1.5\right)$	50% Leverage	3.10 (0.98)	0.53	0.33	0.93	1.5	1.5	1.5

Panel B: Embedded Leverage Using Options: 1986-2012

	Buy and hold	Vol Timing	Vol Timing With Embedded Leverage: Calls	Calls + puts
Sharpe Ratio	0.39	0.59	0.54	0.60
α	–	4.03	5.90	6.67
$s.e.(\alpha)$	–	(1.81)	(3.01)	(2.86)
β	–	0.53	0.59	0.59
Appraisal Ratio	–	0.44	0.39	0.46

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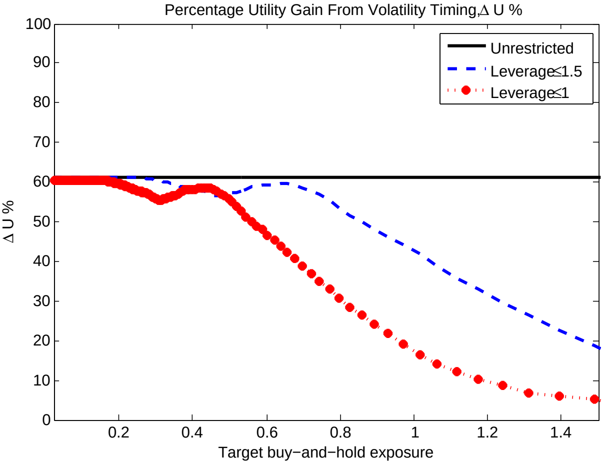
3. Leverage constraints



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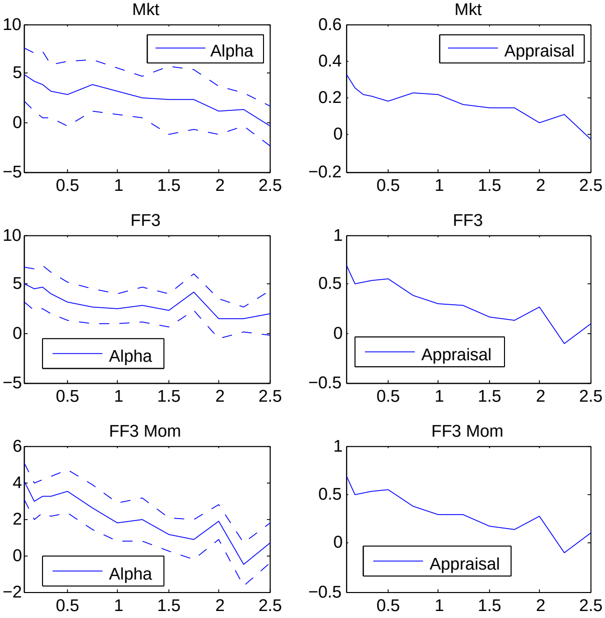
4. Longer holding periods



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Implications

1. Reduced form pricing

   - Risk-adjust mutual fund/ hedge fund strategies

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2. General equilibrium asset pricing models
   - Sharp empirical test of theories of time-varying risk premia
   - GE puzzle: price of risk low when vol is high

Implications

1. Reduced form pricing
   - Risk-adjust mutual fund/ hedge fund strategies
2. General equilibrium asset pricing models
   - Sharp empirical test of theories of time-varying risk premia
   - GE puzzle: price of risk low when vol is high
3. Portfolio choice for long term investors
   - follow up work: we show positive alpha $\Rightarrow$ large wealth gains for both short and long-term oriented investors

     "How Should Investors Respond to Changes in Volatility?" (Moreira Muir)

Conclusion

1. Vol managed portfolios across many factors
   - large α's
   - Large increases in Sharpe ratios
   - Take less risk in recessions and after market crashes
   - Survives transaction costs
   - Works with embedded leverage too
2. Many implications

The dynamics of the risk return tradeoff

Study response to vol shock

$$w_t = \frac{1}{\gamma}\frac{E_t[R_{t+1}]}{Var_t[R_{t+1}]}$$

Vector Auto Regression with $E_t[R_{t+1}]$ and $Var_t[R_{t+1}]$

- Expected returns formed using CAPE and BaaAaa spread
- Expected variance formed using 3 lags of variance in logs

How much should portfolio weight respond?

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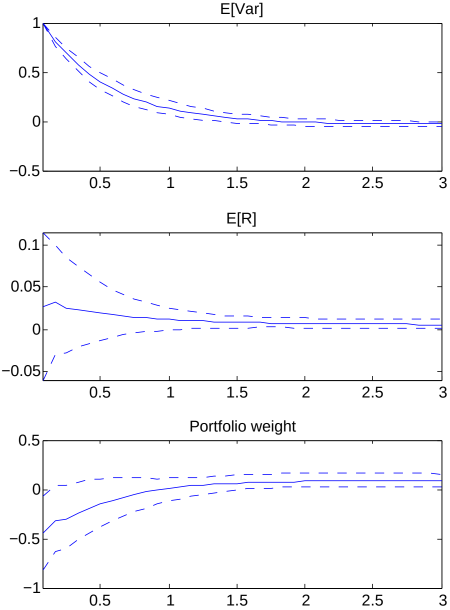
Response to 1 std dev variance shock (in months)



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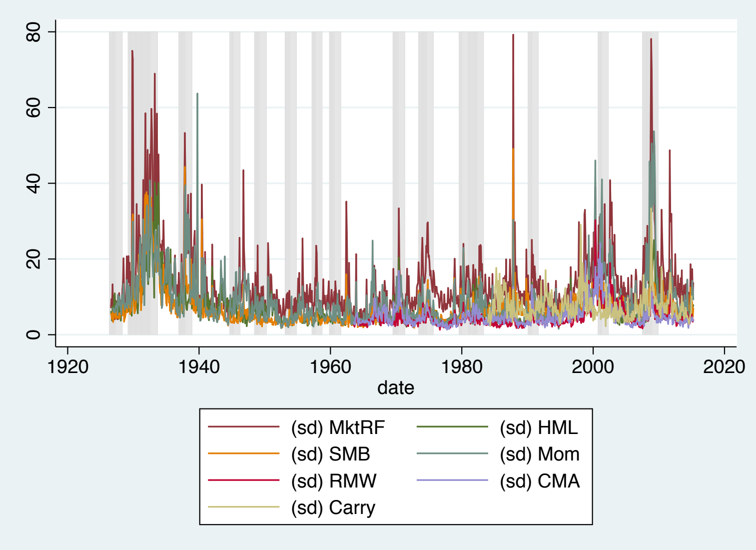
Literature

- Barroso and Santa-Clara (2015) on momentum timing
- Fleming, et al (2001) on volatility timing
  - Daily frequency, cross-sectional, estimate full covariance matrix and assumption about expected returns
- Low risk anomalies in the cross-section: Frazzini and Pedersen (2014), Ang, et. al. (2006)
  - Our portfolios managed in time-series only. Exploit lack of risk-return tradeoff *over time*
- Portfolio choice
  - Viceira and Chacko 2005: Stochastic cash flow vol, expected returns constant in their model
  - Viceira and Campbell 1999, Barberis 2001, etc.: Constant vol, time-varying expected returns

Vol strongly countercyclical for all factors



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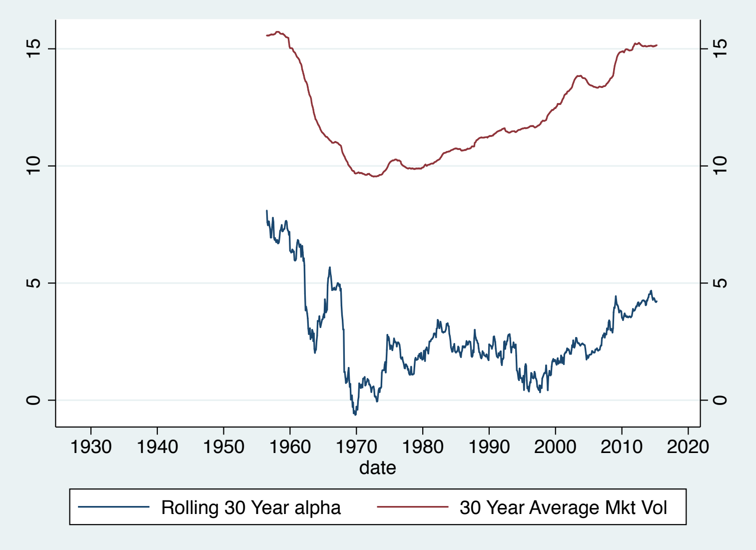
Rolling Alphas



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Does strategy load on the variance risk premium?

- Is strategy highly exposed to a large "surprise" in volatility?
- For Mkt: One-standard deviation increase in variance leads to a 1.4% drop in the buy-and-hold portfolio and only 0.7% drop in the volatility managed counterpart
- Similar for other factors → vol of vol high when vol also high

Moreira and Muir (2015)

Betting against beta

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}	(7) MVE^{σ}
MktRF	0.60						
	(0.05)						
BAB	0.09	0.01	0.02	-0.07	-0.13	-0.06	0.04
	(0.06)	(0.05)	(0.05)	(0.04)	(0.02)	(0.02)	(0.02)
SMB		0.61					
		(0.09)					
HML			0.56				
			(0.07)				
Mom				0.47			
				(0.06)			
RMW					0.65		
					(0.08)		
CMA						0.69	
						(0.04)	
MVE							0.57
							(0.04)
Constant	3.83	-0.77	2.05	13.52	3.97	0.94	4.10
	(1.80)	(1.10)	(1.15)	(1.86)	(0.89)	(0.71)	(0.85)
Observations	996	996	996	996	584	584	996
R-squared	0.37	0.37	0.31	0.21	0.40	0.46	0.33
rmse	52.03	31.36	35.92	51.73	19.95	17.69	26.01

Moreira and Muir (2015)

Risk Parity

Follow Ansess Frazzini Pedersen, construct

$$RP_{t+1} = b_t' f_{t+1} \tag{1}$$

Where

$$b_{i,t}' = \frac{1/\sigma_{i,t}}{\sum 1/\sigma_{i,t}} \tag{2}$$

Control for this factor in our regressions, alphas change very little

Our approach keeps relative weights constant:

$$b_{i,t}' = \frac{b_i}{\sigma_t^2(b' f_{t+1})} \tag{3}$$

Risk-parity factor

Alphas when controlling for risk parity are unchanged

	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom	(8) BAB^{σ}
Alpha (α)	4.86	5.00	4.09	1.32	1.97	2.03	2.38	5.67
	(1.56)	(1.00)	(0.57)	(0.31)	(0.40)	(0.32)	(0.44)	(0.98)
Observations	1,065	1,065	1,060	621	621	575	575	996
R-squared	0.37	0.23	0.26	0.42	0.40	0.50	0.44	0.33
rmse	51.39	34.30	20.25	8.279	9.108	8.497	9.455	29.73

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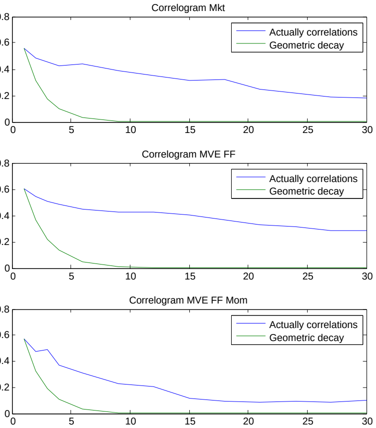
Vol is much more persistent than implied by AR(1) model



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Interpreting results

Define $\gamma_t = \mu_t/\sigma_t^2$. We show:

$$\alpha \quad = \quad -cov\left(\gamma_t, \sigma_t^2\right)\frac{c}{E[\sigma_t^2]}$$

Pricing kernel: Suppose factor f is *conditionally* MVE, then

$$m_{t+1} = \frac{1}{R_{f,t}}\left(1 - \gamma_t\left(f_{t+1} - E_t[f_{t+1}]\right)\right)$$

satisfies

$$E[m_{t+1}R^e_{t+1}] = 0$$

for any excess return R^e.

Thus, if f_{t+1} is conditional pricing factor, then the managed factor $\gamma_t f_{t+1}$ is unconditional pricing factor.

Vector of factors: simplest if orthogonal $\rightarrow \gamma_{i,t} = \mu_{i,t}/\sigma^2_{i,t}$

Interpreting results

Define $\gamma_t = \mu_t/\sigma_t^2$. We show:

$$\alpha = -cov\left(\gamma_t, \sigma_t^2\right) \frac{c}{E[\sigma_t^2]}$$

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Thus, if f_{t+1} is conditional pricing factor, then the managed factor $\gamma_t f_{t+1}$ is unconditional pricing factor.

Vector of factors: simplest case is orthogonal factors, $\gamma_{i,t} = \mu_{i,t}/\sigma^2_{i,t}$

Interpreting results

We show:

$$\alpha \quad = \quad -cov\left(\gamma_t, \sigma_t^2\right)\frac{c}{E[\sigma_t^2]}$$

Positive alpha means negative co-variance

Pricing kernel: Suppose factor f is conditionally MVE, then

$$\frac{d\Pi_t(\gamma_t)}{\Pi_t(\gamma_t)} = -r_t dt - \gamma_t'\left(df_t - E_t\left[df_t\right]\right) \tag{4}$$

satisfies

$$E_t[d(\Pi_t(\gamma_t^*)\widetilde{R})] = 0 \tag{5}$$

for any R. That is, Π_t is valid pricing kernel. γ_t takes conditional pricing factor to unconditional one.

Pricing kernel implications

1. *Assumption*: Factor dF span unconditional MVF for static portfolios of returns $dR = [dR_1, ..., dR_N]'$, and span conditional MVF frontier for dynamic portfolios.

   $\Rightarrow$ Formally: let $\Pi_t(\gamma_t)$ be defined by

$$\frac{d\Pi_t(\gamma_t)}{\Pi_t(\gamma_t)} = -r_t dt - \gamma_t' \left(dF_t - E_t \left[dF_t\right]\right)$$

   then

   - the process $\Pi_t(\gamma_t^*)$ with $\gamma_t^* = \mu_t/\sigma_t^2$ is a valid SDF for dynamics portfolios of dR,
   - the process $\Pi_t(\gamma^u)$ with $\gamma^u = \mu/\sigma^2$ is a valid SDF for static portfolios of dR.

2. *Assumption*: decomposition $\mu_t = b\Sigma_t + \zeta_t$, where $E[\zeta_t|\Sigma_t] = E[\zeta_t]$.

Moreira and Muir (2015)

Pricing kernel implications

1. *Implication*: let $\gamma_t^\sigma = b + \frac{E[\zeta_t]}{\sigma_t^2}$,

   $\Rightarrow$ $\Pi_t(\gamma_t^\sigma)$ is valid SDF for any volatility managed strategy of dR.

2. *Implication*: $Var\left(d\Pi(\gamma_t^*) - d\Pi(\gamma_t^a)\right)$ upper bound on pricing error Sharpe by alternative model γ_t^a

$$
\begin{aligned}
Var\left(d\Pi(\gamma_t^\sigma) - d\Pi(\gamma^u)\right) &= \left(\frac{\alpha}{c}\right)^2 E[\sigma_t^2] J_\sigma^{-1} = 0.11 \\
Var\left(d\Pi(\gamma_t^*) - d\Pi(\gamma^u)\right) &= \left(\frac{\alpha}{c}\right)^2 E[\sigma_t^2] J_\sigma^{-1} + \frac{Var(\zeta_t)}{E[\sigma_t^2]}(J_\sigma + 1) \approx 0.29
\end{aligned}
$$

- note: J_σ is a function of vol of vol

Equilibrium asset pricing models

$$E_t[R_{t+1}] \approx \gamma_t \sigma^2_{R,t}$$

1. **Models:** (i) $cov(E_t[R_{t+1}], \sigma^2_{R,t}) > 0$ and (ii) $cov(\gamma_t, \sigma^2_{R,t}) \geq 0$

   (habits, intermediary, prospect theory, long run risk, rare disasters)

2. Risk-return trade-off literature tests (i): mixed results, low power
   - theory free to ignore evidence

3. We test (ii). In the paper we show that $\alpha > 0 \Rightarrow cov(\gamma_t, \sigma^2_{R,t}) < 0$
   - economically less stringent test
   - econometrically much sharper

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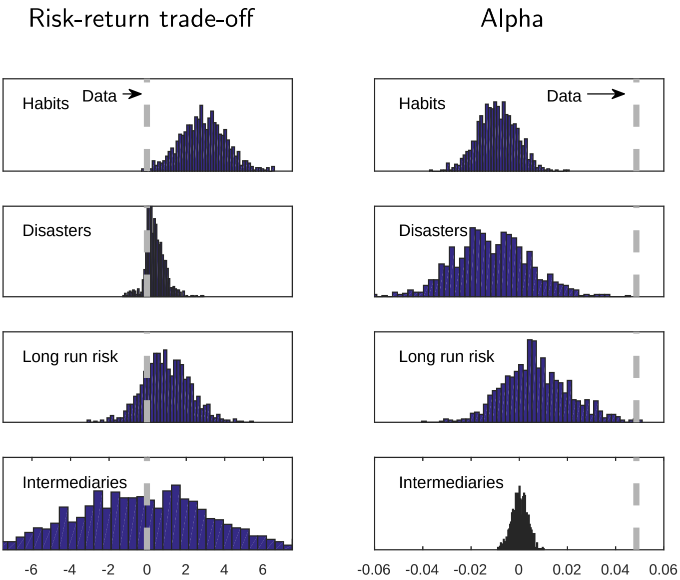
Asset pricing models



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